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Contents

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A brief comparison of interpolation methods for yield curve construction

In our opinion, the following conditions are essential desiderata of a yield curve bootstrap:

(1) all input instruments to the bootstrap are exactly reproduced as outputs of the bootstrap,

(2) the instantaneous forward curve is guaranteed to be positive if the inputs allow it (in particular, the curve is arbitrage free), and

(3) the instantaneous forward curve is typically continuous.

In Hagan and West [2006], Hagan and West [2008] several interpolation methods are considered. All of the methods considered there satisfy the finance exactly, for any input set which is not too dense.

There a new method of interpolation is introduced, the monotone convex method.

To the best of our knowledge the monotone convex method is the first published method where the above criteria are satisfied. In addition, as bonuses

(4) the method is local i.e. changes in inputs at a certain location do not affect in any way the value of the curve at other locations.

(5) the forwards are stable i.e. as inputs change, the instantaneous forwards change more or less proportionately.

(6) hedges constructed by perturbations of this curve are reasonable and stable.

1 Curve Construction Quality Criteria

The criteria to use in judging a curve construction and interpolation should be:

(a) In the case of yield curves, is the curve arbitrage free? Thus we want positivity of the forwards.

(b) In the case of yield curves, how good do the forward rates look? These are usually taken to be the 1m or 3m forward rates, but these are virtually the same as the instantaneous rates. We want as much as possible continuity of the forwards.

(c) How local is the interpolation method? If an input is changed, does the interpolation function only change nearby, with no or minor spill-over elsewhere, or can the changes elsewhere be material?

(d) Are the forwards not only continuous, but also stable? We can quantify the degree of stability by looking for the maximum basis point change in the forward curve given some basis point change (up or down) in one of the inputs. Many of the simpler methods can have this quantity determined exactly, for others we can only derive estimates.

(e) How local are hedges? Suppose we deal an interest rate derivative of a particular tenor. We assign a set of admissible hedging instruments, for example, in the case of a swap curve, we might (even should) decree that the admissible hedging instruments are exactly those instruments that were used to bootstrap the yield curve. Does most of the delta risk get assigned to the hedging instruments that have maturities close to the given tenors, or does a material amount leak into other regions of the curve?

2 A comparison of the methods

Yield curve type	Forwards positive?	Forward smoothness	Method local?	Forwards stable?	Bump hedges local?
Linear on discount	no	not continuous	excellent	excellent	very good
Linear on rates	no	not continuous	excellent	excellent	very good
Raw (linear on log of discount)	yes	not continuous	excellent	excellent	very good
Linear on the log of rates	no	not continuous	excellent	excellent	very good
Piecewise linear forward	no	continuous	poor	very poor	very poor
Quadratic	no	continuous	poor	very poor	very poor
Natural cubic	no	smooth	poor	good	poor
Hermite/Bessel	no	smooth	very good	good	poor
Financial	no	smooth	poor	good	poor
Quadratic natural	no	smooth	poor	good	poor
Hermite/Bessel on rt function	no	smooth	very good	good	poor
Monotone piecewise cubic	no	continuous	very good	good	good
Quartic	no	smooth	poor	very poor	very poor
Monotone convex (unameliorated)	yes	continuous	very good	good	good
Monotone convex (ameliorated)	yes	continuous	good	good	good
Minimal	no	continuous	poor	good	very poor

TABLE 1: *A synopsis of the comparison between methods*

3 Conclusions

It is our opinion that the monotone convex method should be the method of choice for interpolation as it is the only method that we know of which performs well on all the criteria flagged. (In fact, it is the only method which even performs adequately under all the criteria: the fact that it performs well is a bonus.)

In the final analysis, the choice of which method to use will always be subjective, and needs to be decided on a case by case basis. But we hope to have provided some warning flags about many of the methods, and have outlined several qualitative and quantitative criteria for making the selection in which method to use.

References

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