INTEREST RATE DERIVATIVES IN THE SOUTH AFRICAN MARKET BASED ON THE PRIME RATE

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Abstract

Derivatives linked to the prime rate of interest have become quite relevant with the introduction to the exchange traded market of preference shares with payoffs linked to the prime rate. Furthermore, there is an interest in the retail market for retail banks to provide protection to their clients on movements in the prime rate. Here we discuss a possible approach to building models of forward prime and the pricing and hedging of such derivatives. The basic approach is to analyse the statistical cointegration relationship between prime and JIBAR rates.

1. Introduction

Over-the-counter interest rate derivatives comprise instruments such as forward rate agreements, interest rate swaps, caps and floors. These interest rate derivatives set terms for the exchange of cash payments based on changes in market interest rates. Swaps are entered into to change exposure to different areas on a yield curve or to restructure cash flows. In South Africa, vanilla options are typically on the three month JIBAR rate.

On the other hand, interest rate derivatives could also be used to hedge interest rate exposure to the prime interest rate. Customers of retail banks are exposed to the prime interest rate when taking out an overdraft, car loan, or mortgage; and there are debt structures in existence for institutions, which are linked to a spread below the prime rate. Also there are preferential shares in existence which have returns that are prime-linked. Consequently, swaps or options based on mitigating the risk to exposure to the prime interest rate will be of interest to the parties concerned.

2. A model for forward prime

The repurchase (repo) rate is the rate used for borrowing and lending between the South African Reserve Bank and commercial banks. By changing the supply of funds, the SARB can affect short term interest rates (which are determined by the demand and supply of funds), which in turn affects the entire yield curve. This in

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turn affects the overall level of economic activity in the country and hence the general level of prices.

The repo rate is set by Monetary Policy Committee of the South African Reserve Bank. Scheduled MPC meetings occur every two months (other meetings can be convened at short notice); whether or not the repo rate will change, and if so, by how much, is the subject of significant speculation and positioning in financial markets in the time leading up to the announcement. Such has been the procedure since the beginning of 2000. Before then, a rate called SAREP1 played the role of the repo rate. This rate varied daily.

We will need to have a model for forward prime rates. Now, prime is determined by the commercial banks as a trivial function of repo (since late 2000, prime has been 3.50% higher that repo, with some very brief temporary exceptions - never more than a few days when the repo rate has been changed). In principle, prime is a function of the repo rate, so we need a model for forward repo. But JIBAR is also a function of repo, and we clearly have a model for forward JIBAR - this is just the yield curve: forward JIBAR rates are available from elementary calculations with the yield curve that is bootstrapped in the market (Hagan & West 2006). Hence, if we can model prime as a functional form of JIBAR then we will have one of the necessary inputs to our pricing model. This functional form will be a cointegration equation. Essentially, our assumption will be that the functional form that we have seen historically will translate into the same functional form for forward prices.

3. What is cointegration?

Our basic references will be (Harris 1995) and (Alexander 2001).

The usual starting place for asset allocation and risk management is, in the traditional sense from modern portfolio theory, the correlation matrix of returns. In standard models, the price data are differenced before the analysis is even begun, and this differencing typically removes any long-term trends in the data. Of course this data is implicit in the returns data, but under standard models no decisions can be made based upon such trends in the return data.

Roughly, two series are cointegrated if they `move together' (a precise definition will follow). Simple typical examples include an index and its futures price, or the short and long term interest rates. The fundamental aim of cointegration analysis is to detect any common stochastic trends in the price data and to use these common trends for a dynamic analysis for inter-relationships in returns.

Henceforth by `correlation' we mean the correlation of returns. High correlation does not imply high cointegration, or vice virsa. In fact cointegrated series can have correlations that are quite low at times. For example, an index-tracking portfolio should be cointegrated with the index. However, there will on occasion be some stocks which are in the index but not in the portfolio, which have exceptional price movements. Following this, the correlations will be rather low. On the other hand, consider two series, which are both highly cointegrated and highly correlated. Add a drift to the one series. The correlation is almost unaffected, but the cointegration is destroyed. These examples appear in (Alexander 2001).

Thus correlation is not an adequate tool for measuring the long-term behavioural relationship between two instruments. Correlation is intrinsically a short run measure, so correlation based hedging strategies commonly require frequent rebalancing. Such strategies also have no built in mechanism to mitigate basis risk: to ensure the reversion of the hedge to the underlying. Cointegration measures long run co-movements in prices, which may occur even through periods when static correlations appear low.

Although it is perhaps an attractive idea, it would be a mistake to perform time weighted cointegration analysis. The idea behind cointegration is that time series move in conjunction indefinitely. Thus, all data in the sample should bear equal weighting.

Suppose we are performing an analysis of prices of financial instruments. It is standard, but not necessary, to perform cointegration analysis on log prices rather than on prices. In the first instance, it is uncommon to lose statistically significant cointegration in this manner. Secondly, doing so makes the method of analysis consistent with the correlation analysis. Thirdly, error correction models (Alexander 2001, §12.3) have a more natural interpretation when log prices are used. One can argue that when performing cointegration analysis on interest rates, one does not take logarithms: the rate is essentially already the logarithm of price! If the rate is a continuous rate, then the price P and the rate r are related by $P = e^{-r\tau}$, where τ is the term of the rate (and this is fixed in the analysis). If the rate is not continuous, but simple say, then one could either convert to a continuous rate before performing this analysis, or accept that the impact of such a conversion would be immaterial.

A time series is said to be an I(0) process if it is stationary (Alexander 2001, §11.1.2). This means that the random variables have a constant mean, constant variance, and constant lagged covariance structure. A time series is said to be integrated (Alexander 2001, §11.1.3) if the random variable is not stationary, but we get a stationary time series when differencing a finite number of times. The first difference is $\Delta x_t = x_t - x_{t-1}$, the second difference is $\Delta^2 x_t = \Delta x_t - \Delta x_{t-1}$, etc. The smallest number of times differencing needs to occur is called the order of the series; if it is p, then we say the series is I(p).

In the theory of geometric Brownian motion, log prices are integrated of order one. This theoretical concept is borne out empirically.

It is not standard to do ordinary least squares regression on non-stationary data. There is only one circumstance in which regression between integrated variables will give stationary residuals, and that is when the variables are cointegrated. In this case, the regression of the log prices will define their long run equilibrium relationship.

Suppose x_t and y_t are two market variables that are I(1) but not I(0), and there is a β such that $z_t = y_t - \beta x_t$ is I(0), then x_t and y_t are cointegrated and β is the cointegrating factor. This is the Engle-Granger method for testing for cointegration. The basic recipe for analysing Engle-Granger cointegration is as follows:

1. Verify that x_t and y_t are not I(0).
2. Verify that x_t and y_t are I(I).
3. Estimate β by performing an ordinary least squares regression $y_t = \alpha + \beta x_t + \varepsilon_t$.
4. Verify that the residuals ε_t are I(0).

All of the verification above refers to appropriate hypothesis tests. As one can easily see, the fundamental hypothesis test at every stage is testing whether or not some time series is stationary. For this, the Engle-Granger methodology is to use the augmented Dickey-Fuller (ADF) test. In this test, the null hypothesis is that the series is non-stationary while the alternative hypothesis is that it is stationary. For large samples, the critical value at the 5% level for this test is -2.86; at the 1% level it is -3.43. The null hypothesis is rejected if the test statistic is less than the critical value.

Relevant technology for performing cointegration tests is available as a free download at the author's website (West 2007).

If there are more than two instruments to consider in the cointegration analysis, the Engle-Granger method has drawbacks (Alexander 2001, §12.2.2).When n=2, it makes no difference which of the variables is notionally the dependent and which the independent variable in the regression, when we move to more variables, this becomes significant. Remaining with using the Engle-Granger method introduces a statistical bias in the analysis. In this case, the method of (Johansen 1988), (Johansen & Juselius 1990) might be preferable.

Nevertheless, the Engle-Granger methodology is often preferred.

- It is more comprehensible.
- From a financial point of view it has some properties that may be preferable.
- It is usually economically clear what are the dependent and independent variables.
- The fact that in finance one normally has very large samples eliminates most of the biases associated with the multi-factor Engle-Granger methodology.

4. The numerical analysis

4.1 The data

The window of data we use is from Jan 2000, when the current process of intermittant fixing of the Repo rate commenced (doing away with the old daily varying SAREP1 rate) to the time of writing, December 2006. Data is available from http://www.reservebank.co.za/internet/Historicdata.nsf, for example.

4.2. Cleaning the data

First we need to ask if the data is `dirty' in any way. To the best of our knowledge, the only issue was the following: on 28 August 2001 Governor Mboweni announced `a one-off adjustment to the spread between the Reserve Bank's repurchase rate and the interbank call rate by lowering the Reserve Bank’s repurchase rate by 100 basis points with effect from 5 September 2001. The adjustment was purely for administrative reasons and did not imply any change to the monetary policy stance or to any other interest rates' (Casteleijn 2001). In order to clean the data for this event, we simply adjust all the repo rates prior to that event down by 100 points, as if that change had in fact always been in effect.

4.3. Checking the data is I(1)

The prescribed testing method shows that the adjusted repo rate, the prime rate, and the 1m, 3m, 6m and 12m JIBAR rates are I(1). See Table 1. Note that these tests would also work at much, much higher confidence levels.

Table 1: The hypothesis that each series is stationary is rejected; the hypothesis that the first difference of each series is stationary is not rejected. Thus, all the series are accepted as being I(1). Critical value -2.86 at the 5% level and -3.43 at the 1% level.

	adjusted repo	**PRIME**	**1m JIBAR**	**3m JIBAR**	**6m JIBAR**	**12m JIBAR**
data is I(0)	-0,95624	-1,17744	-0,91188	-0,86719	-0,8552	-1,06539
data is I(I)	-41,777447	-41,7765	-36,8974	-38,1235	-37,7867	-41,4102

4.4. JIBAR as a function of repo

In Table 2 we have the relevant statistics for modelling each JIBAR rate as a function of the repo rate. The shorter rates clearly satisfy the cointegration statistic, while the 12m rate satisfies the statistic at the 5% level but not at the 1% level. That the cointegration relationship should deteriorate for longer dated rates is clear: the repo rate is the main signalling device from the Reserve Bank concerning monetary policy, and is most relevant in the short term. In the longer term, other signals will be more important, and the longer term rates reflect an equilibrium view on what the eventual `reality' of that signalling will be.

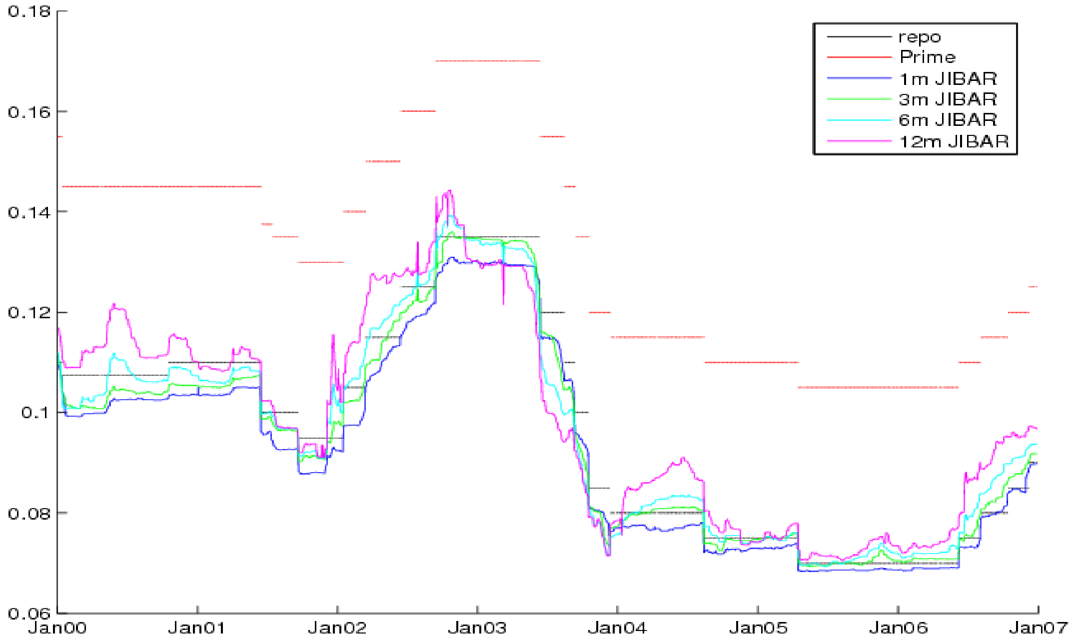

Figure 1: Repo, prime and JIBAR rates

Table 2: Statistics for JIBAR as a function of repo

	1m JIBAR	**3m JIBAR**	**6m JIBAR**	**12m JIBAR**
statistic	-8,1939	-5,9424	-4,1262	-3,1503
R^2	0,9921	0,9823	0,9559	0,8878
α	0,0037	0,0042	0,0063	0,0112
β	0,9195	0,9383	0,9291	0,9061

4.5. Prime as a function of repo or JIBAR

This will be the key functional equation. As discussed previously, prime is a function of repo; we surrogate repo with a suitably chosen JIBAR rate as forward JIBAR is available.

Table 3: Statistics for prime as a function of repo or JIBAR

	repo	**1m JIBAR**	**3m JIBAR**	**6m JIBAR**	**12m JIBAR**
statistic	-19,7493	-8,3097	-6,0779	-4,3531	-3,3160
R^2	0,9965	0,9886	0,9768	0,9513	0,8897
α	0,0346	0,0313	0,0320	0,0325	0,0341
β	1,0078	1,0874	1,0541	1,0362	0,9903

The results are quite remarkable, with an R^2 value of the 1m JIBAR cointegrating equation of 98.9% and the 3m 97.7%. See Table 3.

All of the factors that might make one prefer Engle-Granger to Johansen are relevant here. We know that we want the prime rate to be the dependent variable, and the JIBAR rate or rates to be the independent variable(s). We considered the multi-variable Engle-Granger methodology; it is essentially the same as above; the regression equation is simply a multi-linear equation, with dependent variable the prime rate and independent variables the JIBAR rates. The 6m and 12m JIBAR rates were poor predictors of prime. So, considering a model with 1m and 3m JIBAR, the R^2 only improved marginally, from 98.9% to 99.0%, moreover, the regression is possibly financially unstable, being `long' the 1m rate and `short' the 3m rate. Thus, any hedging strategy based on such a model would be subject to unnecessary twist risk (risk that the two rates will move in an asynchronous way). So, it makes sense simply to stick to the single factor regression model.

Thus, we conclude that an appropriate cointegration equation is the prime rate against the 1m JIBAR rate. In this case, we see that the replicating equation is

$$\text{Prime} = \alpha_1 + \beta_1 J_1 \qquad \ldots (1)$$

where

$\alpha_1 = 0.0313$, $\beta_1 = 1.0874$, and J_m refers to m-month JIBAR. The regression statistic is -8.3097.

At this point we note that it was actually quite fortunate that we did not take logarithms of the rates before running the cointegration analysis. For if we had, our cointegration equation would now be of the form

$$\ln \text{Prime} = \alpha + \beta \ln J$$

$$\text{Prime} = e^{\alpha} J^{\beta}$$

which would be quite unfortunate for the purposes that we have in mind, as will be seen in §5.

We see how the model replicates the prime rate in Figure 2. One should notice how the cointegration smoothes out the jumps to a great extent, as occurs, for example, throughout 2002. This is due to the fact that the market is pricing into the JIBAR rates anticipated jumps to the prime rate before they occur. Only in cases where the quantum of the jump in prime is unexpected in equilibrium - as, for example, in August 2004 and April 2005 - does a jump in JIBAR, and consequentially in the replicating function, occur.

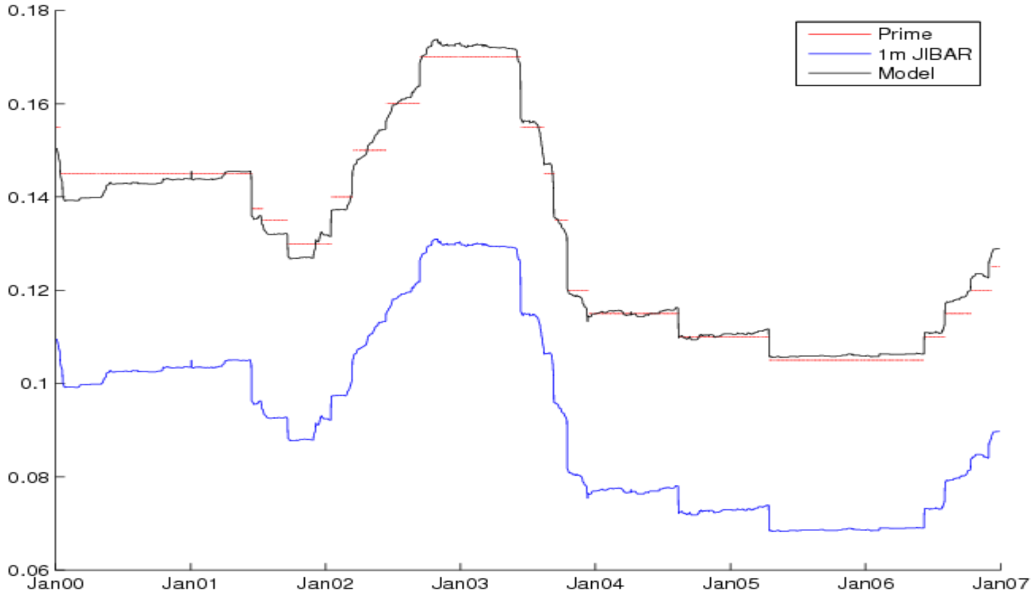

Figure 2: The replication function

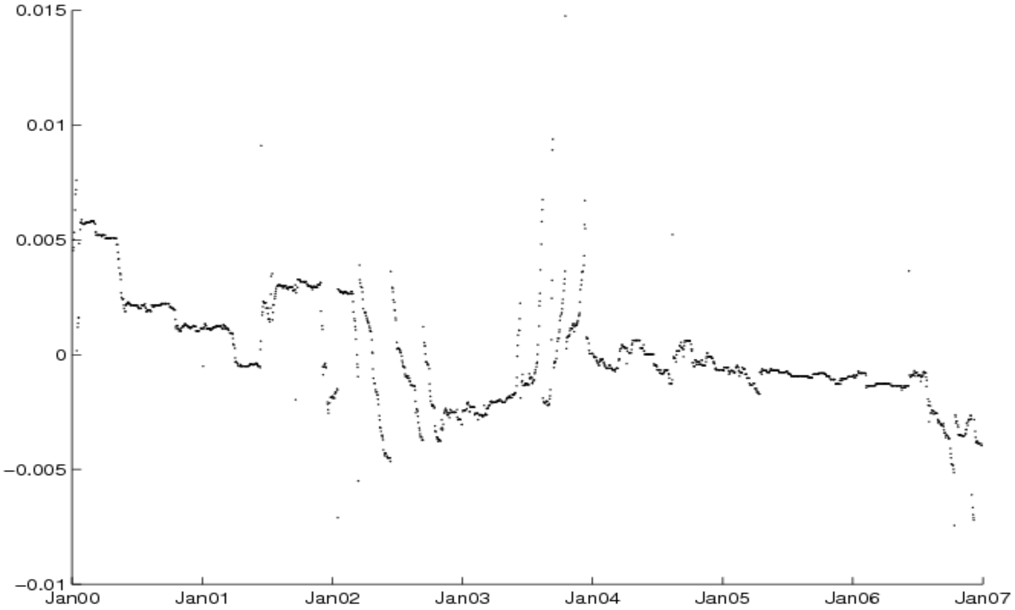

Figure 3: The residuals from the model

5. Pricing prime linked derivatives

The yield curve contains the market equilibrium view on forward JIBAR rates. Our basic thesis is that the cointegration relationship developed in this section will enable us to infer the market equilibrium view on forward prime rates.

We give three examples of pricing derivatives which are linked to prime.

5.1 Bonds with coupons linked to prime

Many of the banks and several corporates issue bonds where the (typically semi-annual) coupon is a percentage (typically in the region of 70%) of the prime rate. These bonds are called preference shares, with the coupon received by the owner of the bond being tax free. Many of these bonds are callable, and some are perpetual. Recently, convertible preference shares have been issued by Anglo Plats and Mvelaphanda (AMSP and MVGP codes) which, if not exercised into shares by the expiry date, convert into such callable perpetual bonds.

How do we value such bonds? We simply bootstrap the yield curve, calculate the simple forward rates for each day in the relevant period, and transform to prime rates by performing the transformation given by the cointegration equation (1). We then value the instrument using these coupons which follow from the contractual specifications. In this world it is important to note that the credit riskiness of the bond issuer is probably a more relevant pricing factor than the stochasticity of interest rates.

5.2. Options on prime

For simplicity we just consider the pricing of a caplet/floorlet on prime under the model of (Black 1976). The relationship between caps and caplets will be similar here as in the usual option market. (Since each caplet in a cap is valued separately we expect a different volatility measure for each.) We use the cointegration equation $P = \alpha_3 + \beta_3 J_3$, using the three month relationship rather than the one month, as it is the three month JIBAR rate which has existing option prices. Thus, we emphasise: the cointegration equation `converts' three month option prices to one month option prices.

Suppose under (Black 1976) the relevant forward JIBAR rate has a volatility measure of $\sigma(J_3)$. Suppose we have a caplet struck at r_X, it pays off if $P > r_X$. Equivalently, if we believe in the cointegration equation, then $P = \alpha_3 + \beta_3 J_3$, and the payoff occurs if $\alpha_3 + \beta_3 J_3 > r_X$, or if $J_3 > \frac{r_X - \alpha_3}{\beta_3}$. Moreover, the payoff is then $\alpha_3 + \beta_3 J_3 - r_X = \beta_3(J_3 - \frac{r_X - \alpha_3}{\beta_3})$. Thus, the caplet is equivalent to β_3-many caplets on JIBAR struck at $\frac{r_X - \alpha_3}{\beta_3}$. Moreover, we easily see that if there is a skew in volatility, then the appropriate volatility to use is that skew volatility at a JIBAR strike of $\frac{r_X - \alpha_3}{\beta_3}$.

Suppose the forward starting date is t_0, one month later is t_1 (so the prime rate on which the option is written applies from t_0 to t_1) and three months later is t_3 (so the JIBAR rate that is being used in the cointegration applies to the period from t_0 to t_3). Then,

$$V = \beta_3 \eta Z(0, t_1) \frac{t_1 - t_0}{365} \left[f(0;t_0,t_3)N(\eta d_1) - \frac{r_X - \alpha_3}{\beta_3} N(\eta d_2) \right]$$

$$d_{1,2} = \frac{\ln \frac{\beta_3 f(0;t_0,t_3)}{r_X - \alpha_3} \pm \frac{1}{2}\sigma(J_3)^2 t_0}{\sigma(J_3)\sqrt{t_0}}$$

where $\eta = 1$ stands for a caplet, $\eta = -1$ for a floorlet, and where the floating rate (swap) curve is being used. $f(0; t_0, t_3)$ is the simple forward rate for the three month period from t_0 to t_3. t_1 is the payment date of the option.

What of the delta of this option? We calculate the delta of this option in units of forward rate agreements on the JIBAR rate. Clearly the delta is given by $\beta_3\eta Z(0,t_1) \frac{t_1 - t_0}{365} N(\eta d_1)$.

An interesting point arises here. The FRA/swap market is the more liquid market, so is the best one available for hedging the derivative instrument. One could argue that the derivative cannot be hedged with positions in prime itself (the bank cannot dynamically adjust their holdings of the actual prime position, as this would involve trading in and out of positions that they have with their retail clients, which is impossible) and so should rather be hedged in the interbank FRA/swap market. In this regard, note if that we assume (as we have done) that the cointegration equation is sacrosanct, then there is no problem. However, if we allow for the possibility of the cointegration equation breaking down, then we have an addition source of randomness in the model. If we accept that hedging these prime derivatives with actual positions in prime is impractical, then the market is now incomplete.

5.3. Prime yield curves

Prime swaps do trade in the market - the floating payments are regular JIBAR payments while the `fixed' payments are a negative spread to the average prime rate in the period. The market settles on this spread. This enables no arbitrage creation of prime yield curves using the techniques developed in (Hagan & West 2006). However, we cannot imagine that this market is in any sort of equilibrium: all commercial banks are receiving prime from their retail customers and so will want to pass it on in a prime swap - but to whom? Thus we feel that the approach that we have developed here has merit because an explicit connection between the prime swap market and the (liquid, two way) JIBAR swap market has been established.

References

Alexander, C. (2001), Market Models: A Guide to Financial Data Analysis, John Wiley and Sons, Ltd.

Black, F. (1976), `The pricing of commodity contracts', Journal of Financial Economics **3**, 167{179.

Casteleijn, A. (2001), South Africa's monetary policy framework, in `Monetary Policy Frameworks in Africa', South African Reserve Bank. **http://www.reservebank.co.za/internet/Publication.nsf/LADV**.

Hagan, P. S. & West, G. (2006), `Interpolation methods for curve construction', Applied Mathematical Finance **13**(2), 89-129.

Harris, R. I. D. (1995), Using cointegration analysis in econometric modelling, Pearson Education.

Johansen, S. (1988), `Statistical analysis of cointegration vectors', Journal of Economic Dynamics and Control **12**, 231-254.

Johansen, S. & Juselius, K. (1990), `Maximum likelihood estimation and inference on cointegration - with applications to the demand for money', Oxford Bulletin of Economics and Statistics **52**(2), 169-210.

West, G. (2007). **http://www.finmod.co.za**.